Game Theory

Lecture 5: applied backward induction

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Dictator game

Player 1 (dictator) divides a pie of $S = 10€$ between herself x_1 and player 2 $x_2 = S - x_1$ in integer values.

What is the Nash equilibrium?

Based on rationality assumption (i.e., pure self interest),

$$x_1^{NE} = S = \pi_1^{NE} \rightarrow \pi_2^{NE} = 0$$

Ultimatum game

Player 1 proposes to divide a pie of $S = 10€$ between herself x_1 and player 2 $x_2 = S - x_1$.

Difference to Dictator game: Payoffs of both players are only realized if player 2 accepts the proposal of player 1.

Rules are *common-knowledge*.

What is the Nash equilibrium?

We can solve this game with **backward induction.**

Ultimatum game

What is the minimum amount x_2^{NE} that player 2 would accept?

→ this maximizes $E(\pi_1)$

Rationality assumption: player 2 is indifferent between accepting or rejecting $x_2 = 0$

→ 50% probability to accept $x_2 = 0$

$$E(\pi_1 | x_2 = 0) = S \times p_{x_2} = 10€ \times 0.5 = 5€$$

→ 100% probability to accept $x_2 > 0$

$$E(\pi_1 | x_2 = 1) = (S - 1) \times p_{x_2} = 9€ \times 1 = 9€$$

In the Nash equilibrium, $x_1^{NE} = S - 1 = \pi_1^{NE} \rightarrow \pi_2^{NE} = 1$

Political conflict game

Player 1 proposes to divide a pie of $S = 10€$ between herself x_1 and player 2 $x_2 = S - x_1$.

Player 2 can accept the proposal or make a counteroffer to divide the discounted value of the second-stage pie

$$\delta S, \text{where } \delta \leq 1.$$

The counteroffer is a split between $\delta S - x_2$ for player 1 and x_2 for player 2.

Player 1 can accept the counteroffer or reject it → a rejection will result in conflict.

In case of conflict, both players have to pay conflicts costs $c_1 = c_2 > 0$, and player 1 wins the second-stage pie δS with probability p_1 and loses with $p_2 = 1 - p_1$. If player 1 loses, player 2 wins.

Backward induction in Political conflict game

In second stage, player 2's rational counteroffer would be an amount $x_1 = \delta S - x_2$ that is equal to player 1's expected payoff in case of conflict $p_1 \delta S - c_1$, assuming that indifference will result in acceptance.

This equation determines player 2's second stage demand: $x_2 = (1 - p_1)\delta S + c_1$

This value of x_2 is what player 2 can expect to earn if play goes to the second stage, so player 1 makes a minimal offer of this amount to player 2 in the first stage:

$$x_1 = S - x_2 = S - (1 - p_1)\delta S - c_1 = (1 - \delta)S + p_1 \delta S - c_1$$

The effects of the payoff parameters are intuitive. As delay costs increase (via a reduction in δ) the initial demand is predicted to increase to take advantage of the strategic first-mover advantage of player 1. One interesting asymmetry for this two-stage game is that the equilibrium demands depend only on the conflict cost for player 1, and that a higher conflict cost of player 1 increases the predicted spread between x_1 and x_2.

Example

If conflict costs are $c_1 = c_2 = 2$, $S = 10$, $\delta = 0.9$, and $p_1 = 0.8$,

Player 1's expected payoffs in case of conflict would be: $p_1 \delta S - c_1 = 5.2$ and

Player 2's expected payoffs in case of conflict would be: $(1 - p_1)\delta S - c_2 = -0.2$.

Then the initial and final demands would be:

$x_1 = 6.2$ and $x_2 = 3.8$.

Since demands have to be integers, x_1 would have to be rounded up. Thus,

$x_1 = 7$ and $x_2 = 3$.

References

- Sieberg, K., Clark, D., Holt, C. A., Nordstrom, T., & Reed, W. (2013). An experimental analysis of asymmetric power in conflict bargaining. *Games, 4*(3), 375-397.