Game Theory

Lecture 5: applied backward induction

Frieder Neunhoeffer



Lisbon School of Economics & Management Universidade de Lisboa





UNIVERSIDADE De lisboa

Dictator game

Player 1 (dictator) divides a pie of $S = 10^{-3}$ in integer values.

What is the Nash equilibrium?

Based on rationality assumption (i.e., pure self interest),

 $x_1^{NE} = S =$



Player 1 (dictator) divides a pie of $S = 10 \in$ between herself x_1 and player 2 $x_2 = S - x_1$

$$=\pi_1^{NE} \rightarrow \pi_2^{NE} = 0$$



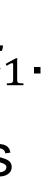
Ultimatum game

- the proposal of player 1.
- Rules are common-knowledge.
- What is the Nash equilibrium?
- We can solve this game with **backward induction**.



Player 1 proposes to divide a pie of $S = 10 \in$ between herself x_1 and player 2 $x_2 = S - x_1$. <u>Difference to Dictator game</u>: Payoffs of both players are only realized if player 2 accepts

Game Theory – Frieder Neunhoeffer



3

Ultimatum game

What is the minimum amount x_2^{NE} that player 2 would accept?

 \rightarrow this maximizes $E(\pi_1)$

<u>Rationality assumption</u>: player 2 is indifferent between accepting or rejecting $x_2 = 0$

- \rightarrow 50% probability to accept $x_2 = 0$
 - $E(\pi_1|x_2=0)=S$
- \rightarrow 100% probability to accept $x_2 > 0$
 - $E(\pi_1|x_2=1)=(S$
- In the Nash equilibrium, $x_1^{NE} = S 1 = \pi_1^N$



$$5 \times p_{x_2} = 10 \in \times 0.5 = 5 \in$$

$$(-1) \times p_{x_2} = 9 \in \times 1 = 9 \in$$

$$_{1}^{NE} \rightarrow \pi_{2}^{NE} = 1$$



Political conflict game

the second-stage pie

The counteroffer is a split between $\delta S - x_2$ for player 1 and x_2 for player 2.

Player 1 can accept the counteroffer or reject it \rightarrow a rejection will result in conflict.

player 2 wins.



- Player 1 proposes to divide a pie of $S = 10 \in$ between herself x_1 and player 2 $x_2 = S x_1$. Player 2 can accept the proposal or make a counteroffer to divide the discounted value of
 - δS , where $\delta \leq 1$.
- In case of conflict, both players have to pay conflicts costs $c_1 = c_2 > 0$, and player 1 wins the second-stage pie δS with probability p_1 and loses with $p_2 = 1 - p_1$. If player 1 loses,







Backward induction in Political conflict game

equal to player 1's expected payoff in case of conflict $p_1 \delta S - c_1$, assuming that indifference will result in acceptance.

This equation determines player 2's second stage demand: $x_2 = (1 - p_1)\delta S + c_1$

This value of x_2 is what player 2 can expect to earn if play goes to the second stage, so player 1 makes a minimal offer of this amount to player 2 in the first stage:

$$x_1 = S - x_2 = S - (1 - p_1)\delta S - c_1 = (1 - \delta)S + p_1\delta S - c_1$$

The effects of the payoff parameters are intuitive. As delay costs increase (via a reduction in δ) the initial demand is predicted to increase to take advantage of the strategic firstmover advantage of player 1. One interesting asymmetry for this two-stage game is that the equilibrium demands depend only on the conflict cost for player 1, and that a higher conflict cost of player 1 increases the predicted spread between x_1 and x_2 .



In second stage, player 2's rational counteroffer would be an amount $x_1 = \delta S - x_2$ that is









Example

If conflict costs are $c_1 = c_2 = 2$, S = 10, $\delta = 0.9$, and $p_1 = 0.8$, Player 1's expected payoffs in case of conflict would be: $p_1 \delta S - c_1 = 5.2$ and Player 2's expected payoffs in case of conflict would be: $(1 - p_1)\delta S - c_2 = -0.2$. Then the initial and final demands would be: $x_1 = 6.2$ and $x_2 = 3.8$. Since demands have to be integers, x_1 would have to be rounded up. Thus,

 $x_1 = 7$ and $x_2 = 3$.





References

analysis of asymmetric power in conflict bargaining. Games, 4(3), 375-397.



• Sieberg, K., Clark, D., Holt, C. A., Nordstrom, T., & Reed, W. (2013). An experimental



